ECE 307 – Techniques for Engineering Decisions

15. Value of Information

George Gross

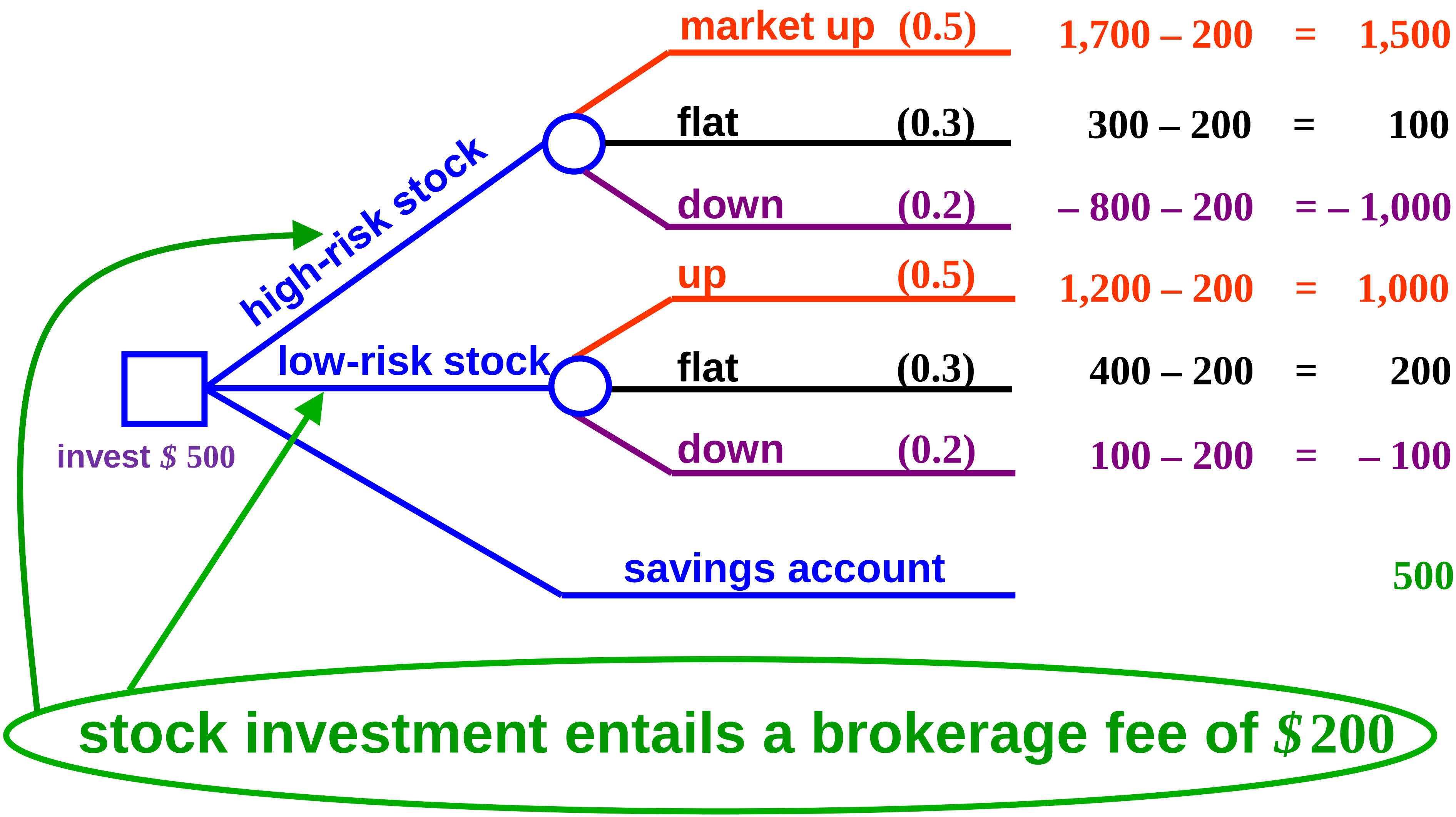
Department of Electrical and Computer Engineering

University of Illinois at Urbana-Champaign

VALUE OF INFORMATION

- While we cannot do away with uncertainty, there is always a desire to attempt to reduce the uncertainty about future outcomes
- The reduction in uncertainty about future outcomes may provide us with choices that strongly *improve the chances* for a good outcome
- We focus on the principles behind information valuation

SIMPLE INVESTMENT EXAMPLE



NOTION OF PERFECT INFORMATION

- **We say that an expert's information is perfect if it is always correct; we may view an expert as a *clairvoyant***

- **We can quantify the value of information in a decision problem by measuring the *expected value of information* (*EVI*)**

NOTION OF PERFECT INFORMATION

- ❑ We consider the role of *perfect information* in the simple investment example
- ❑ In this decision problem, the optimal policy is to invest in high – risk stock since it has the highest returns
- ❑ Suppose an expert predicts that the market goes up: this implies the investor still chooses the high – risk stock investment and consequently the *perfect information* of the expert appears to be of no value

NOTION OF PERFECT INFORMATION

- **On the other hand, suppose the expert predicts a market decrease or a flat market: under this information, the investor's choice is the savings account and the *perfect information* has value because it leads to a *changed* outcome with better performance than would be the case otherwise**
- **In worst case conditions: regardless of the information, we take the same decision as**

NOTION OF PERFECT INFORMATION

without the information and consequently

***EVI = 0*; the interpretation is that we are equally**

well off without the expert

- **Cases in which we have information and we**

  change to a different optimal decision lead to

  ***EVI > 0* since we make a decision that improves**

  the outcome using the available information

EVI ASSESSMENT

- **It follows that the value of information is always nonnegative,** $EVI \geq 0$
- **Indeed,** ***perfect information*** **removes all uncertainty, and the** ***expected value of perfect information EVPI*** **provides an upper bound for** EVI

$$EVI \leq EVPI$$

INVESTMENT EXAMPLE: COMPUTATION OF *EVPI*

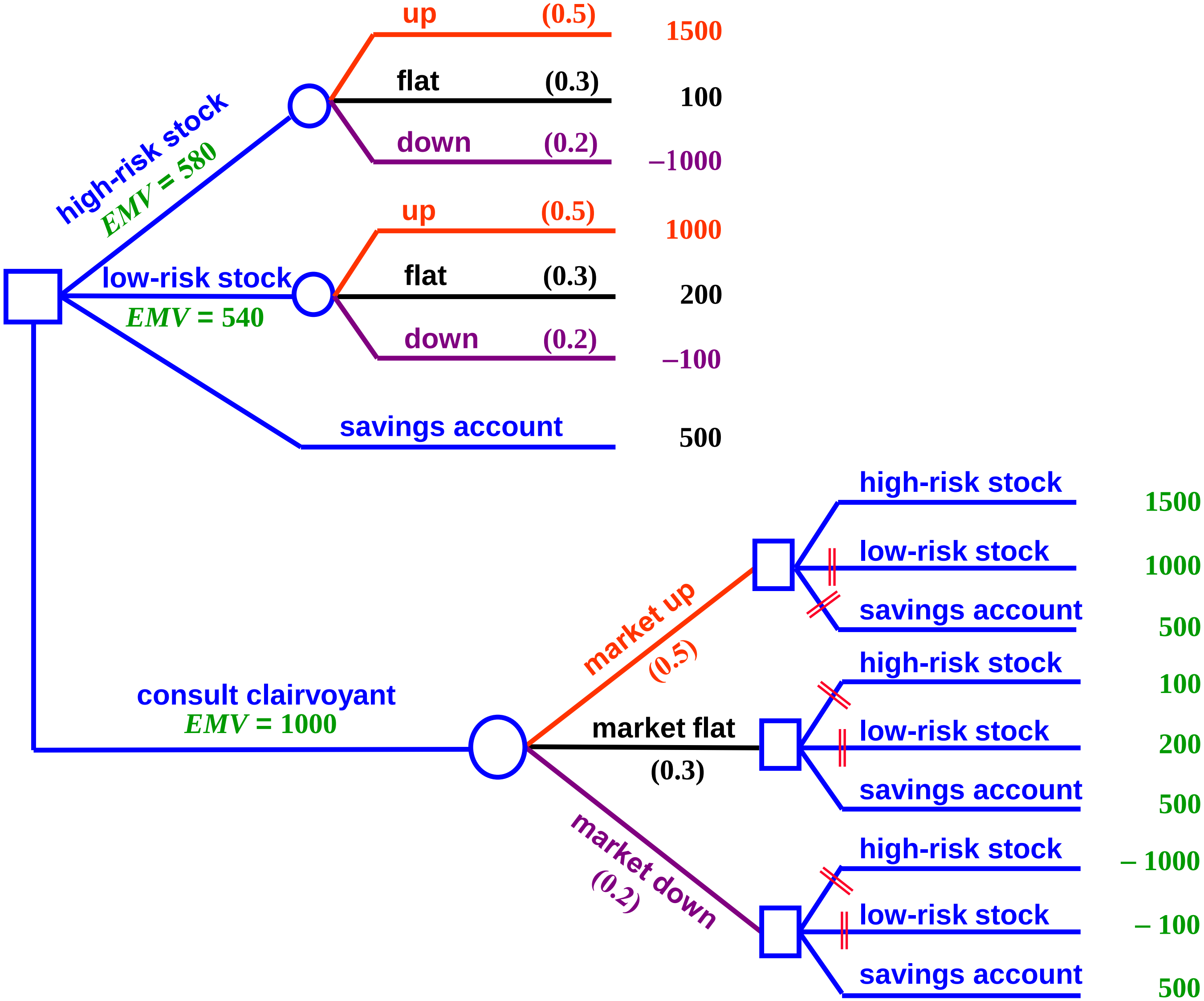
- **Absent any expert information, a value – maximi–zing investor selects the high–risk stock option**
- **The introduction of an expert or clairvoyant brings in *perfect information* since there is perfect *a priori* knowledge of what the market will do before the investor makes his decision and the investor's decision is based on this information**

COMPUTATION OF *EVPI*

- **We use a decision tree approach to compute *EVPI* by reversing the decision and uncertainty order: we view the value of information in an *a priori* sense and define**

$$EVPI = E\left\{\text{decision with perfect information}\right\} - E\left\{\text{decision without additional information}\right\}$$

COMPUTATION OF *EVPI*



COMPUTATION OF *EVPI*

- For the investment problem,

$$EVPI = 1{,}000 - 580 = 420$$

- We may view *EVPI* to represent the maximum amount that the investor is *willing to pay* the expert for the *perfect information* resulting in the improved outcome

EXPECTED VALUE OF IMPERFECT INFORMATION

- **In practice, we cannot obtain *perfect information*; rather, the information is *imperfect* since there are no *clairvoyants***
- **We evaluate the expected value of *imperfect information, EVII***
- **For example, we engage an economist to fore–cast the future stock market trends; the economist's forecasts constitute *imperfect information*: the track record based on past performance is**

EXPECTED VALUE OF IMPERFECT INFORMATION

conditioning event

economist's prediction	*actual market state*		
	up	*flat*	*down*
"*up*"	**0.8**	**0.15**	**0.2**
"*flat*"	**0.1**	**0.7**	**0.2**
"*down*"	**0.1**	**0.15**	**0.6**

conditional probabilities

P{ "flat" | market is flat }

EVII ASSESSMENT

- We use the decision tree approach to compute *EVII*
- For the decision tree, we evaluate probabilities with Bayes' theorem
- For the imperfect information, we define the *r.v.*

$$\underset{\sim}{M} = \begin{array}{c}\text{market}\\ \text{performance}\end{array} = \begin{cases} up & with \;\; probability \;\; 0.5 \\ flat & with \;\; probability \;\; 0.3 \\ down & with \;\; probability \;\; 0.2 \end{cases}$$

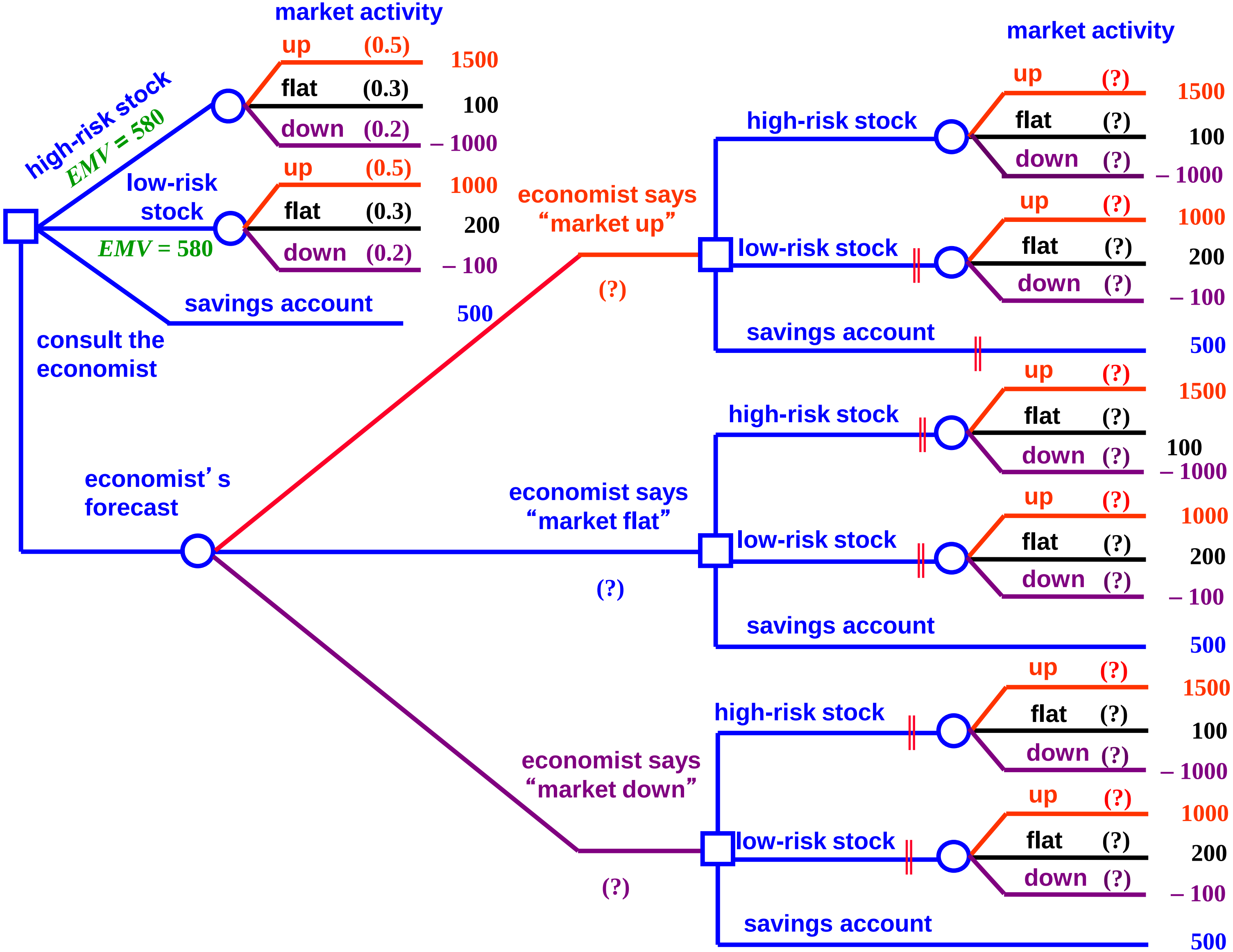
EVII ASSESSMENT

and the forecast *r.v.*

$$\underset{\sim}{F} = \begin{cases} \text{"}up\text{"} \\ \text{"}flat\text{"} \\ \text{"}down\text{"} \end{cases}$$

- ❑ **We have no knowledge of the probabilities of the forecast *r.v.*; all we know is the prior probabilities of $\underset{\sim}{F}$ given $\underset{\sim}{M}$**

EVII COMPUTATION: INCOMPLETE DECISION TREE



COMPUTATION OF REVERSE CONDITIONAL PROBABILITIES

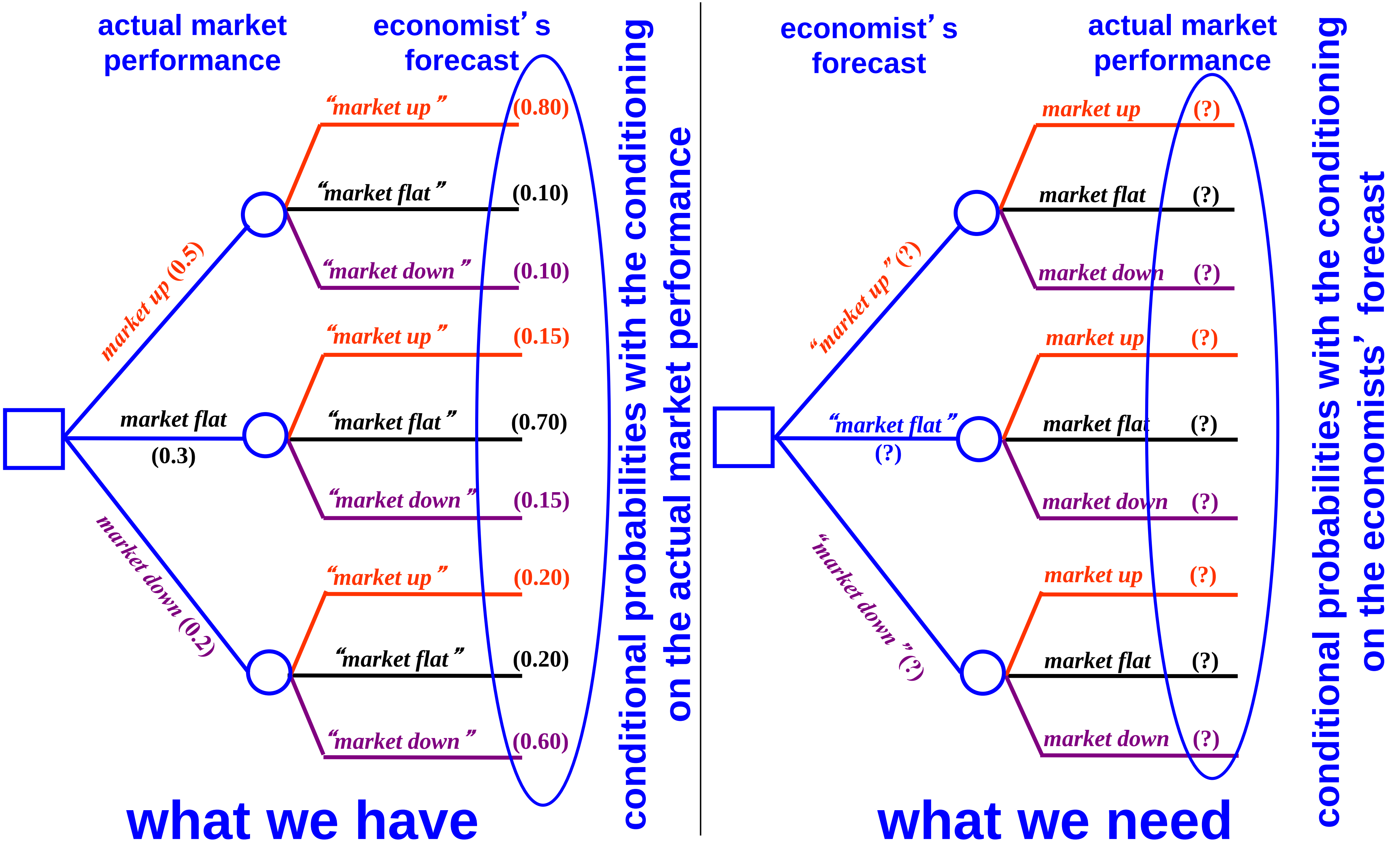
$$P\left\{\underset{\sim}{M} = down \,\middle|\, \underset{\sim}{F} = ''up''\right\} = \frac{P\left\{\underset{\sim}{F} = ''up'' \,\middle|\, \underset{\sim}{M} = down\right\} P\left\{\underset{\sim}{M} = down\right\}}{\left[\begin{array}{l} P\left\{\underset{\sim}{F} = ''up'' \,\middle|\, \underset{\sim}{M} = down\right\} P\left\{\underset{\sim}{M} = down\right\} + \\ P\left\{\underset{\sim}{F} = ''up'' \,\middle|\, \underset{\sim}{M} = up\right\} P\left\{\underset{\sim}{M} = up\right\} + \\ P\left\{\underset{\sim}{F} = ''up'' \,\middle|\, \underset{\sim}{M} = flat\right\} P\left\{\underset{\sim}{M} = flat\right\} \end{array}\right]}$$

$P\left\{\underset{\sim}{F} = ''up''\right\}$ ←

$$= \frac{0.2(0.2)}{0.2(0.2) + 0.15(0.3) + 0.8(0.5)}$$

We *flip* the probabilities in this way

EVII COMPUTATION: FLIPPING THE PROBABILITY TREE



POSTERIOR PROBABILITIES

economist's prediction	*posterior probability for:*		
	market up	***market flat***	***market down***
"*up*"	**0.8247**	**0.0928**	**0.0825**
"*flat*"	**0.1667**	**0.7000**	**0.1333**
"*down*"	**0.2325**	**0.2093**	**0.5581**

conditional probabilities on economists forecast

EVII COMPUTATION

- We use conditional probabilities in the table to build the posterior probabilities
- For example

$$P\left\{ market\ up \middle| economist\ \ predicts\ "up" \right\} = 0.8247$$

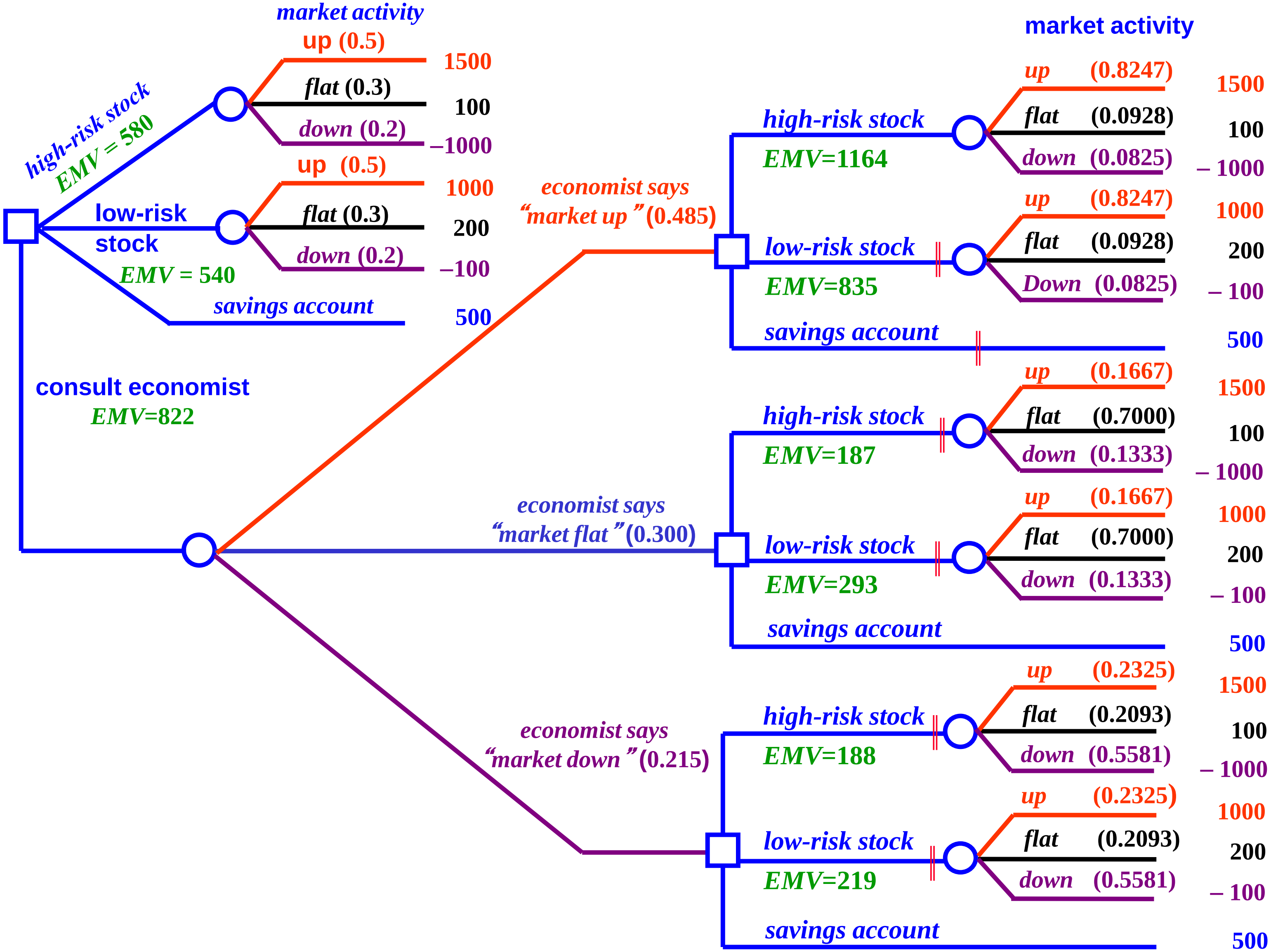
- We then compute

$$P\left\{ \underset{\sim}{F} = "up" \right\} = 0.485$$

$$P\left\{ \underset{\sim}{F} = "flat" \right\} = 0.300$$

$$P\left\{ \underset{\sim}{F} = "down" \right\} = 0.215$$

EXPECTED VALUE OF IMPERFECT INFORMATION



EVII COMPUTATION

- The expected mean value for the decision made with the economist information is

$$EMV\,|_{economist} \;=\; 1{,}164(0.485) + 500(0.515) = 822$$

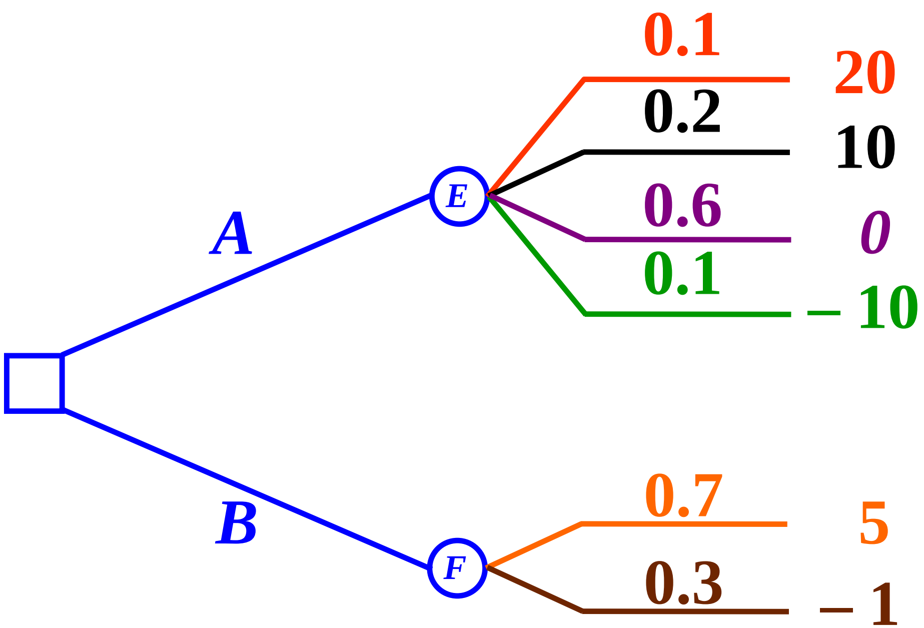
- The expected mean value without information is 580
- Consequently,

$$EVII \;=\; 822 \;-\; 580 \;=\; 242$$

- This value represents the bound on the worth of the economist's forecast

EXAMPLE OF VALUE OF INFORMATION

- We consider the following decision tree

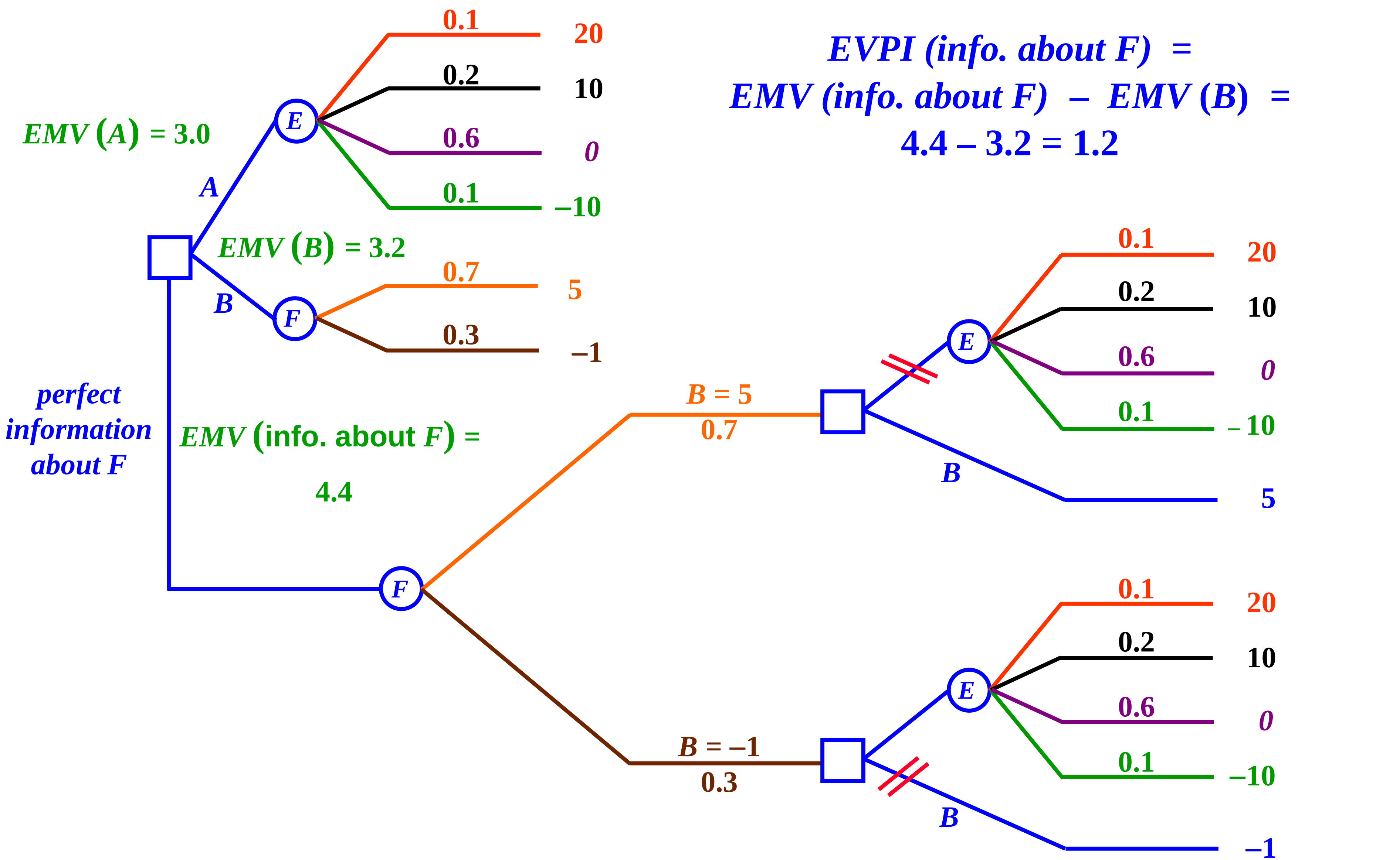


with the events at *E* and *F* as independent

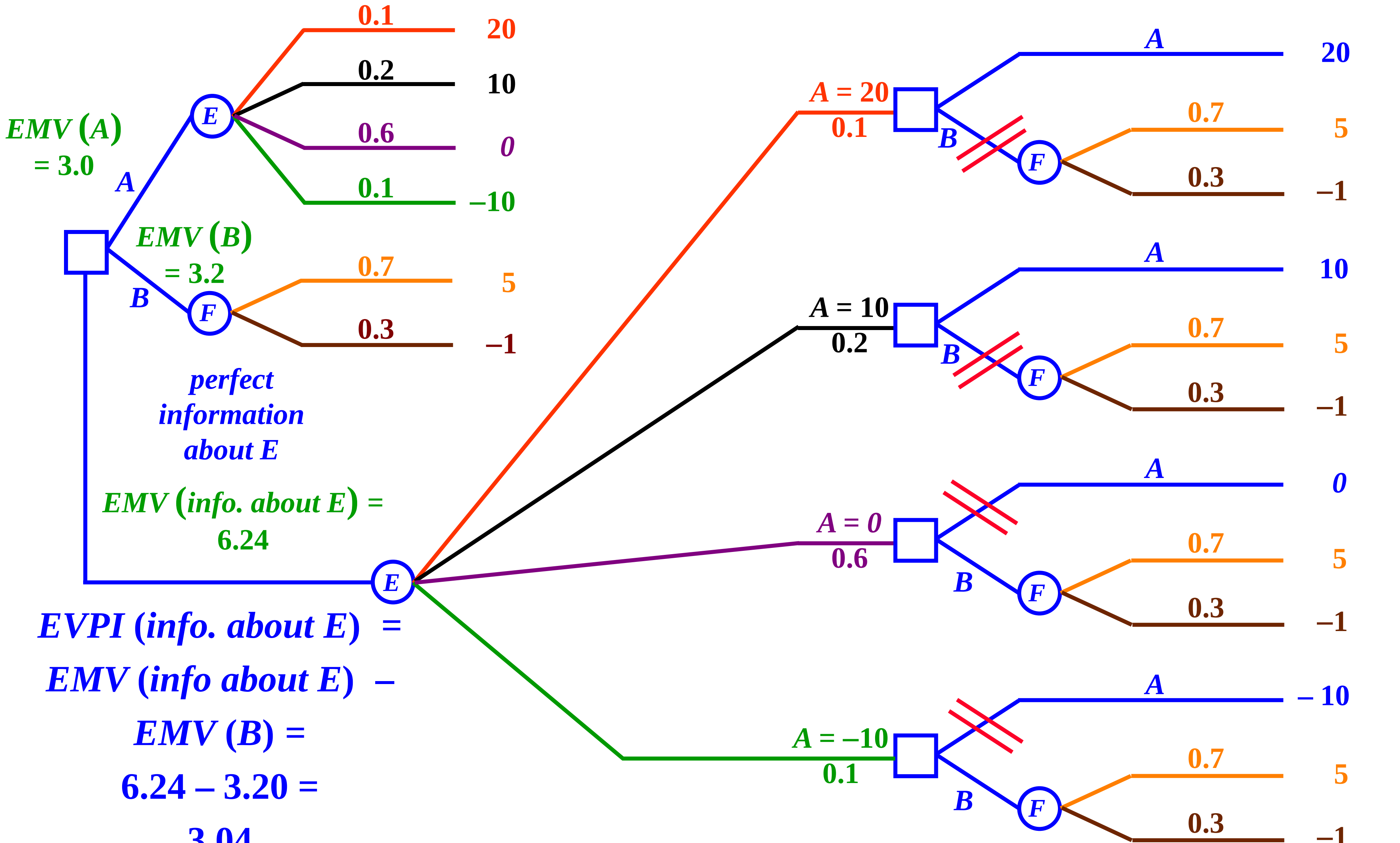
- We perform a number of valuations of *EVPI* for this simple decision problem

EVPI FOR *F* ONLY

EVPI (info. about F) =
EMV (info. about F) – *EMV* (*B*) =
4.4 – 3.2 = 1.2



EVPI FOR *E* ONLY



***EVPI* (*info. about E*) =**

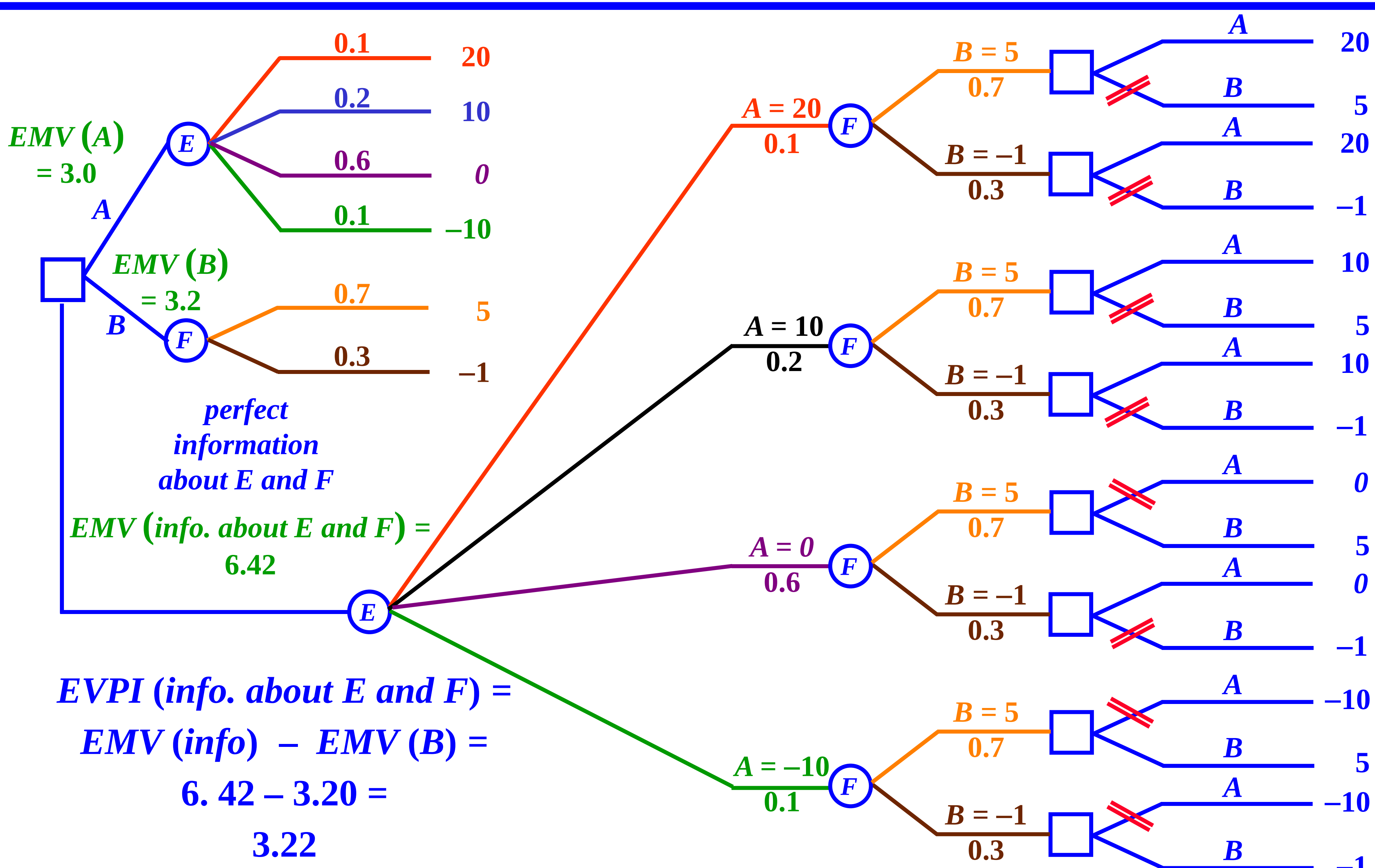
***EMV* (*info about E*) –**

***EMV* (*B*) =**

6.24 – 3.20 =

3.04

EVPI FOR BOTH *E* AND *F*



EVPI (*info. about E and F*) =

EMV (*info*) – *EMV* (*B*) =

6. 42 – 3.20 =

3.22